

Estimating Leaning Tree Failures

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Leaning trees are difficult to assess for catastrophic risk management. In leaning trees, the strength in tension and compression of various portions of the tree above ground, soil strength, and root holding aspects all interact. To better estimate theoretical limits of leaning tree strength, calculations were completed to assist tree specialists appreciate tree biomechanics. These estimates were made under ideal situations for educational purposes. The dynamic loads and unique situations of living trees under natural conditions cannot (and are not) modeled in this publication.

Figure 1 provides the theoretical criteria for the estimates made in this publication. The products produced are: Table 1 -- the degrees from vertical leading to potential failure of trees at various heights under the influence of gravity; and, Table 2 -- a listing of how many feet away from vertical is the tree top for a number of different angles across a number of tree heights.

Table 1: Degrees of angle away from vertical when a tree would potentially fail simply from gravity given its estimated root plate radius (ZRT - zone of rapid taper). The standard root plate value of (0.09 x tree height) is given along with smaller and larger root plate sizes. (Potential failure occurs when $Z > X$.)

ROOT PLATE RADIUS AS % OF TOTAL TREE HEIGHT (X%)	DEGREES OF LEAN FROM VERTICAL CAUSING FAILURE FROM GRAVITY (Y°)
2%	4°
4	8
6	12
8	16
(9) —————	18 -- (standard root plate radius)
10	20
15	31
0.20	42

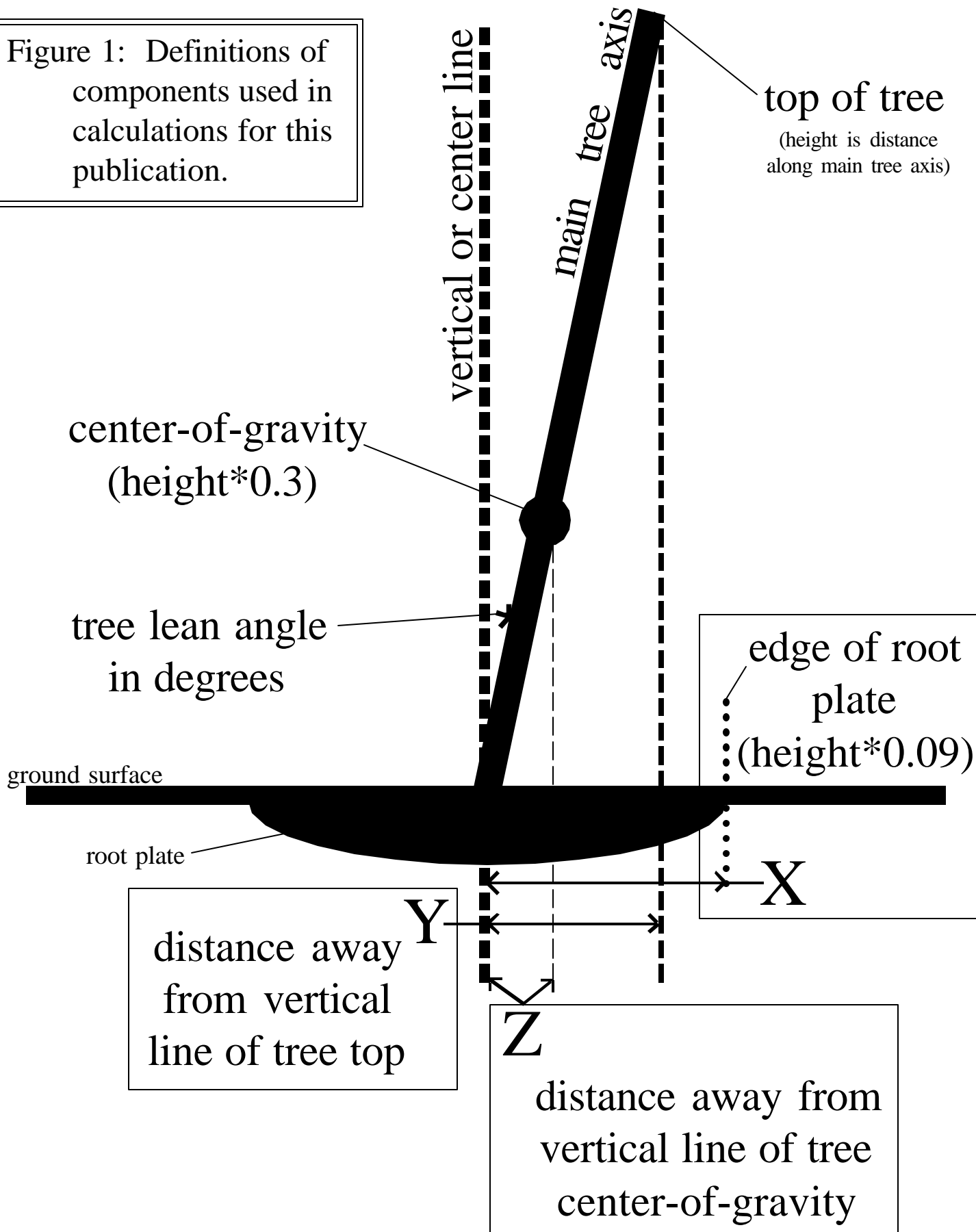


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Table 2: Distance in feet away from vertical of a leaning tree top at a number of different heights. Lean is measured by degrees away from the vertical line anchored at the root collar. Figure 1 graphically defines the problem and results presented here. This table gives the “Y” values from Figure 1.

LEAN DEGREES FROM VERTICAL	TREE HEIGHT ALONG LONGITUDINAL AXIS (FEET)												
	20ft.	30	40	50	60	70	80	90	100	110	120	130	140
1°	0.5ft.	0.5	1	1	1	1	1.5	1.5	2	2	2	2.5	2.5
2	0.5	1	1.5	2	2	2.5	3	3	3.5	4	4	4.5	5
3	1	1.5	2	2.5	3	3.5	4	5	5	6	6	7	7
4	1.5	2	3	3.5	4	5	6	6	7	8	8	9	10
5	2	2.5	3.5	4.5	5	6	7	8	9	10	11	11	12
6	2	3	4	5	6	7	8	9	11	12	13	14	15
7	2.5	3.5	5	6	7	9	10	11	12	13	15	16	17
8	3	4	6	7	8	10	11	13	14	15	17	18	20
9	3	5	6	8	9	11	13	14	16	17	19	20	22
10	3.5	5	7	9	10	12	14	16	17	19	21	23	24
11	4	6	8	10	11	13	15	17	19	21	23	25	27
12	4	6	8	10	13	15	17	19	21	23	25	27	29
13	4.5	7	9	11	14	16	18	20	23	25	27	29	32
14	5	7	10	12	15	17	19	22	24	27	29	31	34
15	5	8	10	13	16	18	21	23	26	29	31	34	36
16	5	8	11	14	17	19	22	25	28	30	33	36	39
17	6	9	12	15	18	21	23	26	29	32	35	38	41
18 - - - -	6 - -	9 - -	12 - -	16 - -	19 - -	22 - -	25 - -	28 - -	31 - -	34 - -	37 - -	40 - -	43 - - -
19	7	10	13	16	20	23	26	29	33	36	39	42	46
20	7	10	14	17	21	24	27	31	34	38	41	45	48
21	7	11	14	18	22	25	29	32	36	39	43	47	50
22	8	11	15	19	23	26	30	34	38	41	45	49	52
23	8	12	16	20	23	27	31	35	39	43	47	51	55
24	8	12	16	20	24	29	33	37	41	45	49	53	57
25	9	13	17	21	25	30	34	38	42	47	51	55	59
26	9	13	18	22	26	31	35	40	44	48	53	57	61
27	9	14	18	23	27	32	36	41	45	50	55	59	64
28	9	14	19	24	28	33	38	42	47	52	56	61	66
29	10	15	19	24	29	34	39	44	49	53	58	63	68
30	10	15	20	25	30	35	40	45	50	55	60	65	70
31	10	15	21	26	31	36	41	46	52	57	62	67	72
32	11	16	21	27	32	37	42	48	53	58	64	69	74
33	11	16	22	27	33	38	44	49	55	60	65	71	76
34	11	17	22	28	34	39	45	50	56	62	67	73	78
35	12	17	23	29	34	40	46	52	57	63	69	75	80
36	12	18	24	29	35	41	47	53	59	65	71	76	80
37	12	18	24	30	36	42	48	54	60	66	72	78	84
38	12	19	25	31	37	43	49	55	62	68	74	80	86
39	13	19	25	32	38	44	50	57	63	69	76	82	88
40	13	19	26	32	39	45	51	58	64	71	77	84	90
41	13	20	26	33	39	46	53	59	66	72	79	85	91
42	13	20	27	34	40	47	54	60	67	74	80	87	94
43	14	21	27	34	41	48	55	61	68	75	82	89	96
44	14	21	28	35	42	49	56	63	70	76	83	90	97
45	14	21	28	35	42	50	57	64	71	78	85	92	99

Figure 1: Definitions of components used in calculations for this publication.



General Calculation Form (See Figure 1 for graphical representation):

Tree Height (length along longitudinal axis)	= TH
Degrees of Lean	= DEGREES
Distance “X” (root plate radius)	= TH x (0.09)
Height of Tree’s Center-of-Gravity	= TH x (0.3)
Distance “Y” (distance away from vertical of tree top)	= TH x (sine of DEGREES)
Distance “Z” (distance away from vertical of tree’s Center-of-Gravity)	= (Height of Tree’s Center-of-Gravity) x (sine of DEGREES)

If “Distance Z” > “Distance X,” then potential exists for failure from gravity.

Conclusions:

Leaning trees are tough to assess and difficult to defend objectively. Theoretical calculations help demonstrate several areas of interest or results. Remember that dynamic loads were not considered and whole tree loss was the only catastrophic failure considered.

The first area of interest is the degrees beyond which gravity works in tandem with wind forces to topple the tree. The actual height of the tree is not important from the standpoint of failure initiation. Height will greatly impact the force with which the tree strikes the ground or target because the force of fall is proportional to tree height to the fifth power (height⁵). As the number of degrees away from vertical (lean angle of the tree) reaches 15°, risk significantly increases of catastrophic failure. As lean angle of the tree passes 20°, structural failure risks are compounded geometrically.

Recommendations should include developing assessments with 5° classes of lean angle for trees in normal use where: 10°-15° requires managerial notice, especially to assure lean is not progressive; 15°-20° removal options considered; 20°-25° removal priority; and, >25° immediate priority removal. The larger the lean the more difficult it is to project dynamic loading and the greater the complexity in defending leaving the tree standing.

Another calculation result is the action of root plates. The larger the root plate radius, the larger the degree of lean sustainable against gravity. The means of trees to increase root plates in special circumstances need further examination. Tree trunk root buttresses, cypress “knees,” and continued root diameter growth in the root collar area all would help increase root plate radius. Root tie-downs and root ball cabling all greatly increase relative root plate radii, and so, allow for greater lean angles. As relative root plate decreases, less lean angle is required to allow gravity effects to become highly significant. Construction damage, close trenching, heavy soil compaction, or other root strength constraining events will lessen the lean angle sustainable.

Table 2 allows estimates in degrees and feet away from vertical of tree tops. By estimating distances from remaining open space, available crown space, witness testimony, and photographic / remote sensing information, Table 2 can help estimate degrees of lean.

Although this treatment deals with whole tree toppling due to lean angle problems, lean assessments need to recognize the additional compression and tension stresses in a leaning tree. Cracks and damage along the neutral plane between compressive forces and tension forces can lead to major losses. On average, trees fail in compression under 2.5 times less force than in tension. Faults on the compressed side of a lean can be compounded greatly. Faults on the tension side of a lean can prevent continued reaction to aspect changes. Species-specific reactions for developing either compression wood or tension wood also must be a consideration.